

Susceptibilities to order α_s in the high density phase of QCD

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Abstract

We compute the free energy density of QCD to order α_s at very high density and non-zero quark masses. The counterterms needed to renormalise the theory to order α_s is same as the vacuum (non-zero density) theory. We investigate the response of the theory to non-zero quark masses and chemical potentials. We study quark number density and quark number susceptibility in the high density limit, where the ratio of the quark mass to the corresponding chemical potential is very small ($m/\mu \ll 1$). In this limit both number density and susceptibility contain a $\ln(m/\mu)$ contribution at order α_s . We compute the scalar and pseudoscalar susceptibilities to order α_s in three flavour QCD at high density taking quark masses and chemical potentials to be degenerate and non-zero. At extremely high density, since α_s is very small, both the susceptibilities are found to be same in the chiral limit. This means that the scalar-pseudoscalar splitting is absent in the CFL phase.

1 Introduction

The study of QCD at very high densities and low temperatures has been received a lot of interest in the last few years. It has been argued that at a sufficiently high density the colour symmetry is spontaneously broken, leading to colour superconductivity [1, 2, 3, 4] (see [5] for recent review). This novel phase might be accessible in the core of neutron stars or in the baryon-rich fragmentation region of heavy-ion collisions.

At extremely high density, where the density is of the order of 3 to 5 times the saturation density of nuclear matter, the wave function of the quarks in nucleons will overlap with that of quarks in other nucleons due to asymptotic freedom. At such high density quarks are no longer confined in nucleons and thus the nuclear matter will become a quark matter [6]. The matter at such high density will consist of a Fermi sea of essentially

free quarks and its behaviour is dominated by the high momentum quarks that live at the Fermi surface. It was shown by Bardeen Cooper and Schriffer (BCS) [7] that in presence of attractive interactions a Fermi surface is unstable. Since in QCD the attractive interaction is provided by one-gluon exchange between quarks in a colour antisymmetric channel, the true ground state of the system is not the naked Fermi surface, but rather a complicated coherent state of particle and hole pairs or condensates, called "Cooper pairs". The formation of condensates leads to energy gaps for both the (quasi) quarks and some or all gluons. At asymptotically high density, and because of asymptotic freedom of QCD, it is possible to compute the quark gap and the gluon masses from first principles [8, 9, 10, 11, 12, 13].

In order to obtain the bulk quantities of a system, such as number density of particles, susceptibilities etc., it is necessary to obtain the free energy density or effective potential of the system. These bulk quantities give us informations about the behaviour of the system in the microscopic level. Freedman and McLerran computed the renormalisation group (RG) improved thermodynamic potential at finite density to order α_s^2 in non-abelian gauge theory with massless quarks [14]. Using this RG improved thermodynamic potential they obtained the effective quark number density. They also obtained using RG equation (Gell-Mann-Low equation) the flow of strong coupling constant as a function of chemical potential. Their evaluation shows that α_s decreases with the increase in density. The two-loop exchange contribution in the thermodynamic potential for massive quarks, later, has been evaluated in the ref.[15]

In the present work we have studied the susceptibilities to leading order in strong coupling constant (order α_s) in the high density phase of QCD. As a first step of this we have computed the free energy density to order α_s at finite density taking quark masses to be non-zero. For massive quarks the cancellation of ultraviolet (UV) divergences is a bit involved issue. The free energy at order α_s is UV divergent. This UV divergent piece has two parts : One is the vacuum (zero density) part and the other one is non-zero density part. However, both the parts get cancelled from the counterterm diagrams. The counterterms needed to make the free energy density finite are the same as what we had in the vacuum theory. Using this renormalized effective action we have computed the quark number density and the quark number susceptibility to order α_s . We find that at finite baryon density, which is high enough too, the density of states of a particular quark flavour near the Fermi surface decreases with the increase in mass of the quark. Therefore, at finite baryon chemical potential the density of states of strange quark is much lesser than that of u and d quarks near the Fermi surface.

The colour and flavour symmetry of QCD with three massless quark flavours break

at high density. The breaking of this flavour and colour symmetry leads to eight massive gluons and gapped (quasi) quarks. The nine (quasi) quarks (three colours times three flavours) fall into an $\mathbf{8} \oplus \mathbf{1}$ of unbroken global SU(3). Therefore, in order to parametrise this phase completely one needs two gap parameters. This particular phase is called the colour-flavour locked (CFL) phase[4]. In the CFL phase both colour and flavour symmetries are dynamically broken by the formation of quark-quark condensate near the Fermi surface. Since the chiral symmetry in this phase is broken as well by the formation of condensates between quark and antiquark pair, the phase can be best described by an effective theory of pseudo-Goldstone bosons [16, 17]. Since the scalar and pseudoscalar susceptibilities are related to the order parameter of the chiral symmetry breaking ($\langle \bar{\psi}\psi \rangle$, where ψ represents the quark field) [18], it is pertinent to study both the susceptibilities in the CFL phase. Using the renormalized free energy density we have computed the scalar and pseudoscalar susceptibilities to order α_s in the high density phase of QCD with three flavours, taking quark masses and chemical potentials to be degenerate and non-zero. We find that at extremely high density, since α_s is very small, both the susceptibilities are same.

We organize the paper as follows. In sec.2 we compute the renormalized free energy density to order α_s . In sec.3 we compute the susceptibilities. In sec.4 we discuss the results. In Appendix-I we give the fermion propagator in the dense medium. In Appendix-II, III and IV we discuss the estimation of integrals which are needed to compute the free energy density in sec.2.

2 Computation of free energy density to order α_s

The gauge Fixed QCD Lagrangian with n_f number of quark flavours at finite density is given as

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_A (i\not{\partial} + g\not{A} - m_A + \mu_A \gamma^0) \psi_A - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2\xi} (\partial^\mu A_\mu^a)^2 \\ & + \partial_\mu c_a^\dagger (\delta^{ab} \partial^\mu - g f^{abc} A^{c\mu}) c_b, \end{aligned} \quad (2.1)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$. a, b, c ($= 1, \dots, 8$) are the gauge group indices and A, B, C ($= 1, \dots, n_f$) are the flavour indices of quarks. ξ is the gauge fixing parameter and we shall take $\xi = 1$ (Feynman gauge) in the following calculations.

2.1 Counterterms to order g^2

At order g^2 the QCD Lagrangian is divergent. The divergences are coming from the one-loop momentum integrals in the quark and gluon two point functions. In order to remove the divergences one needs counterterms. The renormalized QCD Lagrangian reads

$$\mathcal{L}_{ren} = \mathcal{L} + \mathcal{L}_{ct} \quad (2.2)$$

In order to obtain the counterterm Lagrangian let us first calculate the quark two point function at order g^2 (the quark self energy diagram is shown in Fig.1a). We calculate the self energy diagram in Fig.1a using the quark propagator in Appendix-I. This diagram is the sum of two parts: The vacuum part and the finite density part. Since the vacuum part is ultraviolet (UV) finite, we shall calculate only the vacuum part of the quark self energy. The vacuum part of the self energy reads

$$\Sigma^{(vac)}(p)_{AB} = -g^2 C_2(R) \delta_{AB} \Lambda^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} [\gamma^\mu i S(\bar{p} - k) \gamma^\nu] i D_{\mu\nu}(k), \quad (2.3)$$

where the gluon propagator $i D_{\mu\nu}(k) = -\frac{i\eta_{\mu\nu}}{k^2 + i\epsilon}$ (in Feynman gauge), Λ is the regularisation scale and $d (= 4 - 2\epsilon, \epsilon > 0)$ is the space-time dimension. Here $C_2(R) = \frac{4}{3}$ (for SU(3)) and $\bar{p} = (p_0 + \mu_A, \vec{p})$. Performing the k integration in d dimension we obtain

$$\Sigma^{(vac)}(p)_{AB}(p) = ig^2 \delta_{AB} (-1)^{-\epsilon} \frac{\Gamma(\epsilon)}{6\pi^2 (4\pi\Lambda^2)^{-\epsilon}} \int_0^1 dt (1-t)^{-\epsilon} t^{-\epsilon} \frac{(1-\epsilon)t\bar{p} - (2-\epsilon)m_A}{(\bar{p}^2 - \frac{m_A^2}{t})^\epsilon}. \quad (2.4)$$

This term is divergent when $\epsilon \rightarrow 0$. In order to remove this divergence we add to \mathcal{L} the following counterterm (the corresponding counterterm diagram is shown in Fig.1b)

$$\mathcal{L}_{ct}^{(1)} = -\frac{g^2}{6\pi^2 (4\pi\Lambda^2)^{-\epsilon}} (-1)^{-\epsilon} \Gamma(\epsilon) \int_0^1 dt t^{-\epsilon} (1-t)^{-\epsilon} \bar{\psi}_A \mathcal{O} \psi, \quad (2.5)$$

where

$$\mathcal{O} = \frac{(1-\epsilon)t\{\gamma^0(i\partial_0 + \mu_A) - i\vec{\gamma} \cdot \vec{\nabla}\} - m_A(2-\epsilon)}{\left[(i\partial_0 + \mu_A)^2 + \nabla^2 - \frac{m_A^2}{t}\right]^\epsilon}. \quad (2.6)$$

For minimal subtraction we shall retain terms up to zeroth order in ϵ .

Next we calculate the gluon two point function at order g^2 (the gluon polarisation diagram is shown in Fig.2a). Although the diagrams with gluons and the ghosts inside the loops give a non-zero contribution to the gluon polarisation tensor at order g^2 , they contribute zero to the free energy. So for our purpose we calculate the gluon polarisation

taking only quarks in the loop. The vacuum part of the gluon polarisation (since the finite density part is UV finite) reads

$$\Pi_{\mu\nu}^{(vac)ab}(p) = \frac{g^2}{2}\delta^{ab}\Lambda^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} tr_s [iS(k)\gamma^\nu iS(k-p)\gamma^\mu], \quad (2.7)$$

where tr_s means the trace of the product of Dirac gamma-matrices. We evaluate the traces in d space-time dimension using the formulae $\gamma^\mu\gamma_\mu = d$ and $tr_s(\gamma_\mu\gamma_\nu) = f(d)\eta_{\mu\nu}$ ($f(d) \rightarrow 4$ as $d \rightarrow 4$). We drop the terms linear in k and obtain

$$\begin{aligned} \Pi_{\mu\nu}^{(vac)ab}(p) = & -f(d) \int_0^1 dt \Lambda^{2\epsilon} \frac{d^d k}{(2\pi)^d} \\ & \times \frac{2k_\mu k_\nu - \eta_{\mu\nu} k^2 + \eta_{\mu\nu} \{m_A^2 + p^2 t(1-t)\} - 2p_\mu p_\nu t(1-t)}{[k^2 - \{m_A^2 - p^2 t(1-t)\}]^2}, \end{aligned} \quad (2.8)$$

where we have used the Feynman parametrisation. Performing the k -integration in d dimension we obtain

$$\begin{aligned} \Pi_{\mu\nu}^{(vac)ab}(p) = & -ig^2 f(d) \delta^{ab} (-1)^{-\epsilon} \frac{\Gamma(\epsilon)}{16\pi^2 (4\pi\Lambda^2)^{-\epsilon}} \int_0^1 dt t^{-\epsilon} (1-t)^{-\epsilon} \\ & \times \left[\frac{t(1-t)\eta_{\mu\nu}}{\left[p^2 - \frac{m_A^2}{t(1-t)}\right]^{-1+\epsilon}} + \frac{m_A^2 \eta_{\mu\nu} - p_\mu p_\nu t(1-t)}{\left[p^2 - \frac{m_A^2}{t(1-t)}\right]^\epsilon} \right]. \end{aligned} \quad (2.9)$$

This term is divergent as $\epsilon \rightarrow 0$. In order to remove this divergence we add the following counter term to the Lagrangian (the corresponding counterterm diagram is shown in Fig.2b).

$$\mathcal{L}_{ct}^{(2)} = -\frac{g^2}{32\pi^2 (4\pi\Lambda^2)^{-\epsilon}} f(d) (-1)^{-\epsilon} \Gamma(\epsilon) \int_0^1 dt t^{-\epsilon} (1-t)^{-\epsilon} A_\mu^a \mathcal{O}^{\mu\nu} A_\nu^a, \quad (2.10)$$

where

$$\mathcal{O}^{\mu\nu} = \frac{\eta^{\mu\nu} t(1-t)}{\left[-\square - \frac{m_A^2}{t(1-t)}\right]^{-1+\epsilon}} + \frac{m_A^2 \eta^{\mu\nu} + t(1-t) \partial^\mu \partial^\nu}{\left[-\square - \frac{m_A^2}{t(1-t)}\right]^\epsilon}. \quad (2.11)$$

For minimal subtraction we shall retain terms in $\mathcal{L}_{ct}^{(2)}$ upto zeroth order in ϵ . Therefore the renormalized Lagrangian to order g^2 is

$$\mathcal{L}_{ren} = \mathcal{L} + \mathcal{L}_{ct}^{(0)} + \mathcal{L}_{ct}^{(1)} + \mathcal{L}_{ct}^{(2)}, \quad (2.12)$$

where $\mathcal{L}_{ct}^{(0)}$ is a field independent counterterm which is required to renormalise the free (g -independent) part of the free energy density. This can be written as

$$\mathcal{L}_{ct}^{(0)} = \mathcal{C}(m_A, \epsilon), \quad (2.13)$$

where \mathcal{C} is a functions of quark masses (m_A) and ϵ . For minimal subtraction we shall retain in \mathcal{C} the terms upto first order in $\frac{1}{\epsilon}$.

2.2 Free energy density to order α_s

The renormalized QCD partition function reads

$$Z_{ren} = \int \mathcal{D}\phi e^{iS_{ren}(\phi)}, \quad (2.14)$$

where the variable set $\phi \equiv \{\bar{\psi}, \psi, A_\mu^a, c_a^\dagger, c_a\}$. The renormalized free energy density is

$$F = \frac{1}{i\Omega} \ln Z_{ren}, \quad (2.15)$$

where Ω is the 4-space-time volume. Retaining only the g -independent quadratic terms in S_{ren} we expand the remainder up to order g^2 in perturbation theory and integrate over ϕ . The graphs that will contribute to order g^2 are shown in Fig.3. Therefore the renormalized partition function to order g^2 reads

$$Z_{ren} = Z_{ren}^{(0)} \left[1 + \Omega \sum_{A=1}^{n_f} \left\{ \frac{1}{2} I_A^{(3a)} + \frac{1}{2} I_A^{(3b)} + \frac{1}{2} I_A^{(3c)} + I_A^{(3d)} + I_A^{(3e)} + I_A^{(3f)} \right\} \right] \quad (2.16)$$

where $I_A^{(3j)}$ is the contribution of the diagram in Fig.3j ($j = a, b, c, d, e, f$). Here $Z_{ren}^{(0)}$ reads

$$Z_{ren}^{(0)} = e^{i\Omega \mathcal{C}} \prod_{A=1}^{n_f} \left\{ \det[i\cancel{\partial} - m_A + \mu_A \gamma^0 - \epsilon \gamma^0 \partial_0] \right\}^3, \quad (2.17)$$

where the power 3 in the determinant is for three colours associated with each flavour of quarks. The evaluation of this determinant is discussed in the Appendix-II. The final result is

$$\begin{aligned} Z_{ren}^{(0)} = & \exp \left[i\Omega \sum_{A=1}^{n_f} \left[-\frac{3m_A^4}{8\pi^2} \left\{ \frac{3}{2} - \gamma_E - \ln \left(\frac{m_A^2}{4\pi\Lambda^2} \right) \right\} \right. \right. \\ & \left. \left. + \frac{1}{4\pi^2} \left\{ \mu_A \sqrt{\mu_A^2 - m_A^2} \left(\mu_A^2 - \frac{5}{2} m_A^2 \right) + \frac{3}{2} m_A^4 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right\} \right] \right] \end{aligned} \quad (2.18)$$

where we have chosen $\mathcal{C} = \frac{3}{8\pi^2\epsilon} \sum_{A=1}^{n_f} m_A^4$ for minimal subtraction.

We write in the following the contribution of the diagrams in Fig.3.

$$\begin{aligned} I_A^{(3a)} &= -4ig^2 \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{tr_s[iS(p)\gamma^\mu iS(q)\gamma_\mu]}{(p-q)^2}, \\ I_A^{(3b)} &= -4ig^2 \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{A^{\mu\nu\sigma} A_{\mu\nu\sigma}}{p^2 q^2 (p+q)^2}, \\ I_A^{(3c)} &= 24ig^2 \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{p \cdot q}{p^2 q^2 (p-q)^2}, \end{aligned}$$

$$\begin{aligned}
I_A^{(3d)} &= 3ig^2 \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{B}{p^2 q^2}, \\
I_A^{(3e)} &= \frac{ig^2(-1)^{-\epsilon}}{2\pi^2(4\pi\Lambda^2)^{-\epsilon}} \Gamma(\epsilon) \int_0^1 dt t^{-\epsilon} (1-t)^{-\epsilon} \int \frac{d^4 k}{(2\pi)^4} \text{tr}_s[\mathcal{O}(k) iS(k)], \\
I_A^{(3f)} &= -\frac{ig^2(-1)^{-\epsilon}}{4\pi^2(4\pi\Lambda^2)^{-\epsilon}} f(d) \Gamma(\epsilon) \int_0^1 dt t^{-\epsilon} (1-t)^{-\epsilon} \int \frac{d^4 k}{(2\pi)^4} [\mathcal{O}^{\mu\nu}(k) iD_{\mu\nu}(k)], \quad (2.19)
\end{aligned}$$

where

$$\begin{aligned}
A^{\mu\nu\sigma} &= \eta^{\mu\nu}(p-q)^\sigma + \eta^{\nu\sigma}(p+2q)^\mu - \eta^{\sigma\mu}(2p+q)^\nu, \\
B &= (\eta^{\mu\nu}\eta^{\lambda\sigma} - \eta^{\mu\sigma}\eta^{\nu\lambda})(\eta_{\mu\nu}\eta_{\lambda\sigma} - \eta_{\mu\sigma}\eta_{\nu\lambda}), \\
\mathcal{O}(k) &= \frac{(1-\epsilon)t\cancel{k} - m_A(2-\epsilon)}{\left[k^2 - \frac{m_A^2}{t}\right]^\epsilon}, \\
\mathcal{O}^{\mu\nu}(k) &= \frac{\eta^{\mu\nu}t(1-t)}{\left[k^2 - \frac{m_A^2}{t(1-t)}\right]^{-1+\epsilon}} + \frac{m_A^2\eta^{\mu\nu} - t(1-t)k^\mu k^\nu}{\left[k^2 - \frac{m_A^2}{t(1-t)}\right]^\epsilon}. \quad (2.20)
\end{aligned}$$

Only the diagrams in Fig.3a, 3e and 3f will give non-zero contribution to the free energy, the rest will not contribute to the free energy i.e. $I_A^{(3b)} = I_A^{(3c)} = I_A^{(3d)} = 0$. The integral $I_A^{(3a)}$ can be written as the sum of three terms : The vacuum part, density dependent UV divergent part and density dependent finite part. So we write it as

$$I_A^{(3a)} = 4ig^2[f(d)I_A^{(0)} + f(d)I_A^{(1)} + I_A^{(2)}], \quad (2.21)$$

where we have evaluated the trace over the Dirac gamma matrices in d space-time dimension and write the d -dimensional integrals in the following:

$$\begin{aligned}
I_A^{(0)} &= \Lambda^{4\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d p}{(2\pi)^d} \frac{(2-d)k.p + m_A^2 d}{(k^2 - m_A^2 + i\epsilon)(p^2 - m_A^2 + i\epsilon)\{(p-k)^2 + i\epsilon\}}, \\
I_A^{(1)} &= 4\pi i \int \frac{d^4 k}{(2\pi)^4} \delta(k^2 - m_A^2) \theta(k_0) \theta(\mu_A - k_0) \Lambda^{2\epsilon} \int \frac{d^d p}{(2\pi)^d} \\
&\quad \times \frac{(2-d)k.p + m_A^2 d}{(k^2 - m_A^2 + i\epsilon)(p^2 - m_A^2 + i\epsilon)\{(p-k)^2 + i\epsilon\}}, \\
I_A^{(2)} &= -16\pi^2 \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \delta(k^2 - m_A^2) \delta(p^2 - m_A^2) \theta(k_0) \theta(p_0) \theta(\mu_A - k_0) \theta(\mu_A - p_0) \\
&\quad \times \frac{2m_A^2 - k.p}{m_A^2 - k.p}. \quad (2.22)
\end{aligned}$$

The integral $I_A^{(3e)}$ can be splitted up into two parts: The vacuum part and the density dependent UV divergent part. Going over to d space-time dimension and evaluating the

spinor traces we can write it as

$$I_A^{(3e)} = \frac{ig^2}{2\pi^2} f(d) [J_A^{(0)} + J_A^{(1)}], \quad (2.23)$$

where

$$\begin{aligned} J_A^{(0)} &= \frac{i(-1)^{-\epsilon}}{(4\pi\Lambda^2)^{-\epsilon}} \Gamma(\epsilon) \int_0^1 dt t^{-\epsilon} (1-t)^{-\epsilon} \Lambda^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{(1-\epsilon)tk^2 - m_A^2(2-\epsilon)}{(k^2 - m_A^2 + i\epsilon) \left[k^2 - \frac{m_A^2}{t} \right]^\epsilon}, \\ J_A^{(1)} &= -m_A^2 \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-\epsilon} \frac{\Gamma(\epsilon)}{1-2\epsilon} \left[\frac{1}{2-2\epsilon} - (2-\epsilon) \right] \\ &\quad \times \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k^2 - m_A^2) \theta(k_0) \theta(\mu_A - k_0). \end{aligned} \quad (2.24)$$

In a similar fashion we can write $I_A^{(3f)}$, going over to d space-time dimension as

$$I_A^{(3f)} = -\frac{ig^2}{4\pi^2} f(d) K_A^{(0)}, \quad (2.25)$$

where

$$K_A^{(0)} = -\frac{i(-1)^{-\epsilon}(d-1)}{(4\pi\Lambda^2)^{-\epsilon}} \Gamma(\epsilon) \int_0^1 dt t^{1-\epsilon} (1-t)^{1-\epsilon} \Lambda^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{\left[k^2 - \frac{m_A^2}{t(1-t)} \right]^\epsilon}. \quad (2.26)$$

In the minimal subtraction scheme we shall retain terms in $J_A^{(0)}$, $J_A^{(1)}$ and $K_A^{(0)}$ upto first order in $\frac{1}{\epsilon}$. The evaluation of the integrals $I_A^{(0)}$, $I_A^{(1)}$, $I_A^{(2)}$, $J_A^{(0)}$ and $J_A^{(1)}$ are given in the Appendix (III and IV). The results are written in the following.

$$\begin{aligned} I_A^{(0)} &= \left(\frac{m_A^2}{16\pi^2} \right)^2 \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-2\epsilon} \left[-\frac{3}{\epsilon^2} - \frac{7}{\epsilon} + \frac{6\gamma_E}{\epsilon} - 15 + 14\gamma_E - 6\gamma_E^2 - \frac{\pi^2}{2} \right], \\ I_A^{(1)} &= -\frac{m_A^2}{64\pi^4} \left[\mu_A \sqrt{\mu_A^2 - m_A^2} - m_A^2 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right] \\ &\quad \times \left\{ \frac{3}{\epsilon} \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-\epsilon} + 4 - 3\gamma_E \right\}, \\ I_A^{(2)} &= -\frac{1}{16\pi^4} (\mu_A^2 - m_A^2)(\mu_A^2 - 2m_A^2) + \frac{3m_A^2}{8\pi^4} \mu_A \sqrt{\mu_A^2 - m_A^2} \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \\ &\quad - \frac{m_A^2 \mu_A^2}{2\pi^4} \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) + \frac{m_A^4}{16\pi^4} \ln^2 \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \\ &\quad - \frac{m_A^2 (\mu_A - m_A)^2}{4\pi^4} \ln \left(\frac{\sqrt{\mu_A + m_A} + \sqrt{\mu_A - m_A}}{2\sqrt{\mu_A}} \right), \end{aligned}$$

$$\begin{aligned}
J_A^{(0)} &= \frac{m_A^4}{32\pi^2} \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-2\epsilon} \left[\frac{3}{\epsilon^2} + \frac{7}{\epsilon} - \frac{6}{\epsilon} \gamma_E \right], \\
J_A^{(1)} &= \frac{3m_A^2}{16\pi^2\epsilon} \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-\epsilon} \left[\mu_A \sqrt{\mu_A^2 - m_A^2} - m_A^2 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right], \\
K_A^{(0)} &= \frac{m_A^4}{32\pi^2} \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-2\epsilon} \left[\frac{3}{\epsilon^2} + \frac{7}{\epsilon} - \frac{6}{\epsilon} \gamma_E \right].
\end{aligned} \tag{2.27}$$

The vacuum UV divergences in the diagram Fig.3a get cancelled from the counterterm diagrams Fig.3e and 3f and the density dependent UV divergence in Fig.3a gets cancelled from the counterterm diagram Fig.3e. Therefore the renormalized free energy density to order α_s reads

$$F = \sum_{A=1}^{n_f} f_A, \tag{2.28}$$

where

$$\begin{aligned}
f_A &= -\frac{3m_A^4}{8\pi^2} \left[\frac{3}{2} - \gamma_E - \ln \left(\frac{m_A^2}{4\pi\Lambda^2} \right) \right] \\
&+ \frac{1}{4\pi^2} \left\{ \mu_A \sqrt{\mu_A^2 - m_A^2} \left(\mu_A^2 - \frac{5}{2} m_A^2 \right) + \frac{3}{2} m_A^4 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right\} \\
&+ \frac{\alpha_s}{\pi^3} \left[-\frac{m_A^4}{8} \left(15 - 14\gamma_E + 6\gamma_E^2 + \frac{\pi^2}{2} \right) - \frac{1}{2} (\mu_A^2 - m_A^2) (\mu_A^2 - 2m_A^2) \right. \\
&- \left(2 - \frac{3}{2} \gamma_E \right) m_A^2 \mu_A \sqrt{\mu_A^2 - m_A^2} + \frac{m_A^4}{2} \ln^2 \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \\
&+ m_A^2 \left\{ \left(2 - \frac{3}{2} \gamma_E \right) m_A^2 + 3\mu_A \sqrt{\mu_A^2 - m_A^2} - 4\mu_A^2 \right\} \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \\
&\left. - 2m_A^2 (\mu_A - m_A)^2 \ln \left(\frac{\sqrt{\mu_A + m_A} + \sqrt{\mu_A - m_A}}{2\sqrt{\mu_A}} \right) \right] + 0(\alpha_s^2).
\end{aligned} \tag{2.29}$$

3 Response to chemical potentials and non-zero quark masses

In this section we shall investigate the response of the system with the change in chemical potentials and quark masses. The quark number density is obtained by taking a derivative of the free energy density with respect to chemical potential. In the high density limit this is given by

$$n_A = \frac{\partial F}{\partial \mu_A}$$

$$\begin{aligned}
&= \frac{\mu_A^3}{\pi^2} - \frac{3}{2\pi^2} m_A^2 \mu_A + \frac{\alpha_s}{\pi^3} \left[-2\mu_A^3 + (8 + 3\gamma_E) m_A^2 \mu_A - 4m_A^3 - 14m_A^2 \mu_A \ln \left(\frac{m_A}{2\mu_A} \right) \right] \\
&\quad + 0(\alpha_s, \frac{1}{\mu_A}).
\end{aligned} \tag{3.1}$$

Here we have retained terms upto zero-th order in μ_A . In the chiral limit with non-interacting quarks the chemical potential $\mu_A \sim n_A^{\frac{1}{3}}$. However in the non-chiral limit with interacting quarks the chemical potential cannot take such a simple form in terms of density.

The quark number susceptibility to order α_s in the high density limit ($\frac{m_A}{\mu_A} \ll 1$) reads

$$\begin{aligned}
\chi_A &= \frac{\partial n_A}{\partial \mu_A} \\
&= \frac{3}{\pi^2} \mu_A^2 - \frac{3}{2\pi^2} m_A^2 + \frac{\alpha_s}{\pi^3} \left[-6\mu_A^2 + 3(4 + \gamma_E) m_A^2 - 14m_A^2 \ln \left(\frac{m_A}{2\mu_A} \right) \right] \\
&\quad + 0(\alpha_s^2, \frac{1}{\mu_A}),
\end{aligned} \tag{3.2}$$

where we have retained terms upto zero-th order in μ_A . The susceptibility has a simple physical interpretation. It measures the density of states of a particular quark flavour near the Fermi surface. At finite baryon density ($\mu_A = \bar{\mu}$ for all A , where $\bar{\mu}$ is the baryon chemical potential) as the mass of the fermion increases the susceptibility or the density of states near the Fermi surface decreases. Therefore the density of states of the strange quark near the Fermi surface at finite baryon density is smaller compared to the u and d quarks.

The colour and flavour symmetry of QCD with three massless quark flavours break at high density. This breaking leads to eight massive gluons and gapped (quasi) quarks. This particular phase is called CFL phase [4]. We shall calculate in this phase the scalar and pseudoscalar susceptibilities which are related to the chiral order parameter of the theory. In order to do this we consider the three flavour QCD with u , d and s quarks at high density. We take quark masses and chemical potentials to be degenerate; $m_u = m_d = m_s = m$ and $\mu_u = \mu_d = \mu_s = \mu$. This means that the theory respect the $SU(3)_{L+R}$ flavour symmetry. Let us define the following scalar and pseudoscalar correlations:

$$\begin{aligned}
C_S(q) &= \int d^4x e^{iq \cdot x} \langle 0 | \bar{\psi}(x) \psi(x) \bar{\psi}(0) \psi(0) | 0 \rangle, \\
C_P(q) &= \int d^4x e^{iq \cdot x} \langle 0 | \bar{\psi}(x) i\gamma_5 \psi(x) \bar{\psi}(0) i\gamma_5 \psi(0) | 0 \rangle.
\end{aligned} \tag{3.3}$$

The scalar and pseudoscalar susceptibilities are defined as,

$$\chi_S = C_S(0) \quad \text{and} \quad \chi_P = C_P(0). \tag{3.4}$$

These two susceptibilities are related to the chiral order parameter in the following way [18]

$$\chi_S = \frac{\partial \langle \bar{\psi}\psi \rangle}{\partial m} \quad \text{and} \quad \chi_P = \frac{\langle \bar{\psi}\psi \rangle}{m}, \quad (3.5)$$

where $\langle \bar{\psi}\psi \rangle = \langle \bar{u}u + \bar{d}d + \bar{s}s \rangle$. The chiral condensate is obtained by differentiating the free energy density with respect to m . Once we obtain $\langle \bar{\psi}\psi \rangle$ we can compute the scalar and pseudoscalar susceptibilities using eqn.(3.5). Both the scalar and pseudoscalar susceptibilities in the high density limit read

$$\begin{aligned} \chi_S &= -\frac{9m^2}{2\pi^2} \left[1 - 3\gamma_E - 3 \ln \left(\frac{m^2}{4\pi\Lambda^2} \right) \right] + \frac{3}{\pi^2} \left[\mu^2 - 3m^2 - \frac{9}{2}m^2 \ln \left(\frac{m}{2\mu} \right) \right] \\ &\quad + \frac{3\alpha_s}{\pi^3} \left[-(13 - 3\gamma_E)\mu^2 - \left\{ 19 - \frac{39}{2}\gamma_E + 9\gamma_E^2 + \frac{3}{4}\pi^2 \right\} m^2 \right. \\ &\quad \left. + (18\gamma_E - 5)m^2 \ln \left(\frac{m}{2\mu} \right) + 6m^2 \ln^2 \left(\frac{m}{2\mu} \right) + 8\mu^2 \ln \left(\frac{m}{2\mu} \right) \right] + 0(\alpha_s^2, \frac{1}{\mu}), \\ \chi_P &= -\frac{9m^2}{2\pi^2} \left[1 - \gamma_E - \ln \left(\frac{m^2}{4\pi\Lambda^2} \right) \right] + \frac{3}{\pi^2} \left[\mu^2 - \frac{m^2}{2} - \frac{3}{2}m^2 \ln \left(\frac{m}{2\mu} \right) \right] \\ &\quad + \frac{3\alpha_s}{\pi^3} \left[(8 - 3\gamma_E)\mu^2 - \left\{ 10 - \frac{17}{2}\gamma_E + 3\gamma_E^2 + \frac{1}{4}\pi^2 \right\} m^2 \right. \\ &\quad \left. - (1 - 6\gamma_E)m^2 \ln \left(\frac{m}{2\mu} \right) + 2m^2 \ln^2 \left(\frac{m}{2\mu} \right) + 2\mu^2 \ln \left(\frac{m}{2\mu} \right) \right] + 0(\alpha_s^2, \frac{1}{\mu}). \quad (3.6) \end{aligned}$$

At very high density ($\frac{m}{\mu} \ll 1$) as we take the chiral limit ($m \rightarrow 0$) both the susceptibilities are logarithmically divergent at order α_s , i.e. $\chi_S \sim \frac{3}{\pi^2}\mu^2 \left\{ 1 - \frac{\alpha_s}{\pi}(13 - 3\gamma_E) + \frac{8\alpha_s}{\pi} \ln \left(\frac{m}{2\mu} \right) \right\}$ and $\chi_P \sim \frac{3}{\pi^2}\mu^2 \left\{ 1 + \frac{\alpha_s}{\pi}(8 - 3\gamma_E) + \frac{2\alpha_s}{\pi} \ln \left(\frac{m}{2\mu} \right) \right\}$. Since at very high density $\alpha_s \ll 1$ [14], the resulting contribution at order α_s is small compared to the leading order (zero-th order in α_s) one. Therefore, at extremely high density both scalar and pseudoscalar susceptibilities are same in the chiral limit, i.e. $\chi_S = \chi_P = \frac{3}{\pi^2}\mu^2$.

4 Discussion and Conclusions

We have computed the free energy density of QCD to order α_s at finite density taking quark masses to be non-zero. The free energy density is found to UV divergent. The counterterms needed to renormalise it is the same as what we had in the vacuum theory. We have computed the quark number density and the quark number susceptibility in the high density limit ($m_A/\mu_A \ll 1$) using this free energy density. In this limit both of them contain a $\ln(m_A/\mu_A)$ contribution at order α_s .

We have computed the scalar and pseudoscalar susceptibility in the high density phase of three flavour QCD taking masses and chemical potentials of the quarks to be degenerate and non-zero. We find that at very high density, since $\alpha_s \ll 1$, both the susceptibilities are same in the chiral limit. Since the mass is related to the susceptibility via $M^2 = Z\chi^{-1}$ [19], where Z is the wavefunction renormalisation, and since the scalars and pseudoscalars renormalise in the same way, we obtain the following equality in the chiral limit: $M_S^2 = M_P^2$. Therefore in the three flavour QCD both the scalar and pseudoscalar masses are degenerate in the CFL phase.

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Appendix-I: Fermion propagator at finite density

Free fermion propagator at finite density reads

$$\begin{aligned} iS(x) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \frac{i}{\not{p} - m - \mu\gamma^0 + i\epsilon\gamma^0 p_0} \\ &= i \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \frac{\gamma^0(p_0 + \mu) - \vec{\gamma} \cdot \vec{p} + m}{(p_0 + \mu + i\epsilon p_0)^2 - \omega_p^2}, \end{aligned} \quad (\text{I.1})$$

where $\omega_p = \sqrt{\vec{p}^2 + m^2}$. Performing the p_0 integration we obtain

$$\begin{aligned} iS(x) &= \theta(x^0) \int \frac{d^3p}{(2\pi)^3} \frac{\not{p} + m}{2p_0} \theta(p_0 - \mu) e^{-ip \cdot x + i\mu x^0} \\ &\quad \theta(-x^0) \int \frac{d^3p}{(2\pi)^3} \left[\frac{\not{p} + m}{2p_0} \theta(\mu - p_0) e^{-ip \cdot x + i\mu x^0} + \frac{\not{p} - m}{2p_0} e^{ip \cdot x + i\mu x^0} \right], \end{aligned} \quad (\text{I.2})$$

where $p_0 = \omega_p$. After doing some algebra we can write it as

$$iS(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x + i\mu x^0} iS(p), \quad (\text{I.3})$$

where $iS(p) = (\not{p} + m)i\tilde{S}(p)$ and

$$i\tilde{S}(p) = \frac{i}{p^2 - m^2 + i\epsilon} - 2\pi\delta(p^2 - m^2)\theta(p_0)\theta(\mu - p_0). \quad (\text{I.4})$$

Appendix-II: Evaluation of $Z_{ren}^{(0)}$

Define

$$D = i\rlap{\not{D}} - m_A + \mu_A \gamma^0 - \epsilon \gamma^0 \partial_0. \quad (\text{II.1})$$

Now

$$\det D = e^{Tr \ln D}, \quad (\text{II.2})$$

where Tr is taken over spin matrices and space-time.

$$Tr \ln D = \int d^4x \langle x | tr_s \ln D | x \rangle. \quad (\text{II.3})$$

Inserting complete set of eigenstates of momentum in between and using the formulae $\langle k | x \rangle = e^{-ik \cdot x}$, $\langle k' | k \rangle = (2\pi)^4 \delta^4(k - k')$ we obtain

$$Tr \ln D = \int d^4x \frac{d^4k}{(2\pi)^4} tr_s \ln D(k). \quad (\text{II.4})$$

Here we have used the fact that operator D is diagonal in the momentum representation i.e. $\langle k | D | k' \rangle = (2\pi)^4 \delta^4(k - k') D(k)$. Using the formula $tr \ln D = \ln \det D$ we obtain

$$Tr \ln D = \int d^4x \frac{d^4k}{(2\pi)^4} \ln \det_s D(k), \quad (\text{II.5})$$

where the \det_s means the determinant of the resulting spin matrix. Evaluating the determinant of the spin matrix we obtain

$$Tr \ln D = 2 \int d^4x \frac{d^4k}{(2\pi)^4} \ln(\bar{k}^2 - m_A^2), \quad (\text{II.6})$$

where $\bar{k} = (k_0 + \mu_A + i\epsilon k_0, \vec{k})$. Therefore the renormalized free partition function reads,

$$\begin{aligned} Z_{ren}^{(0)} &= e^{i\Omega C} \prod_{A=1}^{n_f} \{\det D\}^3 \\ &= e^{i\Omega C} \exp \left\{ 6\Omega \sum_{A=1}^{n_f} \int \frac{d^4k}{(2\pi)^4} \ln(\bar{k}^2 - m_A^2) \right\}. \end{aligned} \quad (\text{II.7})$$

We use the following parametrisation

$$\ln(\bar{k}^2 - m_A^2) = \int_0^{\mu_A + \omega_{kA}} \frac{dx}{k_0 + (1 - i\epsilon)x} + \int_0^{\mu_A - \omega_{kA}} \frac{dx}{k_0 + (1 - i\epsilon)x}, \quad (\text{II.8})$$

where $\omega_{kA} = \sqrt{\vec{k}^2 + m_A^2}$. Here we have ignored the mass(m_A) and density(μ_A) independent terms. We first do the k_0 -integration going over to the d space time dimension and

then do the integration over x . Next we do the remaining k integrations in $d - 1$ and 3 dimension for the vacuum and the finite density part respectively. The result is

$$\begin{aligned} \Lambda^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \ln(\bar{k}^2 - m_A^2) &= -\frac{im_A^4}{16\pi^2} \left[\frac{1}{\epsilon} + \frac{3}{2} - \gamma_E - \ln \left(\frac{m_A^2}{4\pi\Lambda^2} \right) \right] \\ &+ \frac{i}{24\pi^2} \left[\mu_A \sqrt{\mu_A^2 - m_A^2} \left(\mu_A^2 - \frac{5}{2} m_A^2 \right) \right. \\ &\left. + \frac{3}{2} m_A^4 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right]. \end{aligned} \quad (\text{II.9})$$

Appendix-III: Evaluation of $I_A^{(0)}$, $I_A^{(1)}$ and $I_A^{(2)}$

Performing the momentum integration over k and p the integral $I_A^{(0)}$ reads

$$\begin{aligned} I_A^{(0)} &= \left(\frac{m_A^2}{16\pi^2} \right)^2 \left(\frac{m_A}{4\pi\Lambda^2} \right)^{-2\epsilon} (2 - \epsilon) \Gamma(-1 + 2\epsilon) \int_0^1 dy y^{-2+\epsilon} \\ &\times \left\{ (2y - 1) \int_0^1 dx x^{-\epsilon} (1 - xy)^{-2+\epsilon} + (1 - y) \int_0^1 dx x^{-\epsilon} (1 - xy)^{-3+\epsilon} \right\}. \end{aligned} \quad (\text{III.1})$$

Define

$$I_n = \int_0^1 dx x^{-\epsilon} (1 - xy)^{-n} \quad (n \geq 0), \quad (\text{III.2})$$

where it satisfies the following recursion relation

$$I_n = \frac{1}{1 - \epsilon} [(1 - y)^{-n} + n(I_n - I_{n+1})]. \quad (\text{III.3})$$

Using this recursion relation we obtain,

$$I_{2-\epsilon} = \frac{(1 - y)^{-1+\epsilon}}{1 - \epsilon}, \quad I_{3-\epsilon} = \frac{(1 - y)^{-2+\epsilon}}{2 - \epsilon} + \frac{(1 - y)^{-1+\epsilon}}{(1 - \epsilon)(2 - \epsilon)}. \quad (\text{III.4})$$

Therefore we can write

$$\begin{aligned} I_A^{(0)} &= \left(\frac{m_A^2}{16\pi^2} \right)^2 \left(\frac{m_A}{4\pi\Lambda^2} \right)^{-2\epsilon} (2 - \epsilon) \Gamma(-1 + 2\epsilon) \int_0^1 dy y^{-2+\epsilon} \\ &\times \{ (2y - 1) I_{2-\epsilon} + (1 - y) I_{3-\epsilon} \}. \end{aligned} \quad (\text{III.5})$$

Using eqn.(III.4) and performing the integration over y we obtain

$$I_A^{(0)} = \left(\frac{m_A^2}{16\pi^2} \right)^2 \left(\frac{m_A}{4\pi\Lambda^2} \right)^{-2\epsilon} \left[-\frac{3}{\epsilon^2} - \frac{7}{\epsilon} + \frac{6}{\epsilon} \gamma_E - 15 + 14\gamma_E - 6\gamma_E^2 - \frac{\pi^2}{2} \right]. \quad (\text{III.6})$$

Performing the integration over p the integral $I_A^{(1)}$ reads

$$\begin{aligned}
I_A^{(1)} &= \frac{m_A^2}{8\pi^2} \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-\epsilon} \left[-\frac{3}{\epsilon} - 4 + 3\gamma_E \right] \\
&\quad \times \int \frac{d^3k}{(2\pi)^3 2\omega_{kA}} \theta(\mu_A - \omega_{kA}) \\
&= \frac{m_A^2}{64\pi^4} \left(\frac{m_A^2}{4\pi\Lambda^2} \right)^{-\epsilon} \left[-\frac{3}{\epsilon} - 4 + 3\gamma_E \right] \\
&\quad \times \left[\mu_A \sqrt{\mu_A^2 - m_A^2} - m_A^2 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right], \tag{III.7}
\end{aligned}$$

where

$$\int \frac{d^3k}{(2\pi)^3 2\omega_{kA}} \theta(\mu_A - \omega_{kA}) = \frac{1}{8\pi^2} \left[\mu_A \sqrt{\mu_A^2 - m_A^2} - m_A^2 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right]. \tag{III.8}$$

The integral $I_A^{(2)}$ reads

$$I_A^{(2)} = -\frac{1}{16\pi^4} \left[\mu_A \sqrt{\mu_A^2 - m_A^2} - m_A^2 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right] + \frac{m_A^2}{8\pi^4} L_A, \tag{III.9}$$

where

$$L_A = 2\mu_A^2 \left[(1 - \alpha)^2 \ln \alpha + \int_{\alpha}^1 dx \int_{\alpha}^1 dy \left\{ \ln |x - y| - \ln |xy + \sqrt{x^2 - \alpha^2} \sqrt{y^2 - \alpha^2} - \alpha^2| \right\} \right] \tag{III.10}$$

and $\alpha = m_A/\mu_A$. We can carry out the first integral and the result is

$$\int_{\alpha}^1 dx \int_{\alpha}^1 dy \ln |x - y| = (1 - \alpha)^2 \ln(1 - \alpha) - \frac{3}{2}(1 - \alpha)^2. \tag{III.11}$$

In order to do the second integral we change the variables from x and y to θ and ϕ respectively where $x = \alpha \cosh \theta$ and $y = \alpha \cosh \phi$. The integral becomes

$$\begin{aligned}
&\int_{\alpha}^1 dx \int_{\alpha}^1 dy \ln |xy - \sqrt{x^2 - \alpha^2} \sqrt{y^2 - \alpha^2} - \alpha^2| \\
&= 2(1 - \alpha)^2 \ln \alpha + (1 - \alpha)^2 \ln 2 + 2\alpha^2 \int_0^u d(\cosh \theta) \int_0^u d(\cosh \phi) \\
&\quad \times \ln \sinh \left(\frac{\theta + \phi}{2} \right), \tag{III.12}
\end{aligned}$$

where $u = \ln \left(\frac{1 + \sqrt{1 - \alpha^2}}{\alpha} \right)$. The remaining integration over θ and ϕ can be done easily. Finally the result for L_A is

$$L_A = \mu_A^2 - m_A^2 - 4\mu_A^2 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right)$$

$$\begin{aligned}
& +2\mu_A\sqrt{\mu_A^2 - m_A^2} \ln\left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A}\right) + m_A^2 \ln^2\left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A}\right) \\
& -2(\mu_A - m_A)^2 \ln\left(\frac{\sqrt{\mu_A + m_A} + \sqrt{\mu_A - m_A}}{2\sqrt{\mu_A}}\right). \tag{III.13}
\end{aligned}$$

Appendix-IV: Evaluation of $J_A^{(0)}$ and $J_A^{(1)}$

Performing the integration over k the integral $J_A^{(0)}$ reads

$$J_A^{(0)} = -\frac{m_A^4}{16\pi^2} \left(\frac{m_A}{4\pi\Lambda^2}\right)^{-2\epsilon} \Gamma(-2+\epsilon)(1-\epsilon)(2-\epsilon)[X_1 - X_2], \tag{IV.1}$$

where

$$X_1 = \int_0^1 dy y^{-\epsilon} (1-y)^{-1+\epsilon} \int_0^1 dx \frac{x^{-1+\epsilon}}{(1-y+xy)^{-2+2\epsilon}} \tag{IV.2}$$

$$X_2 = 2 \int_0^1 dy y^{-\epsilon} (1-y)^{-1+\epsilon} \int_0^1 dx \frac{x^{-1+\epsilon}}{(1-y+xy)^{-1+2\epsilon}} \tag{IV.3}$$

Let us first evaluate X_1 . Performing a integration by parts with respect to x we obtain,

$$\begin{aligned}
X_1 = & \int_0^1 dy y^{-\epsilon} (1-y)^{-1+\epsilon} \left[\frac{1}{\epsilon} - \frac{2(1-\epsilon)}{\epsilon(1+\epsilon)} y \right. \\
& \left. + \frac{2(1-\epsilon)(1-2\epsilon)}{\epsilon(1+\epsilon)} y^2 \int_0^1 dx x^{1+\epsilon} (1-y+xy)^{-2\epsilon} \right]. \tag{IV.4}
\end{aligned}$$

There remains a integration over x within the third bracket. This integrand is finite at $\epsilon = 0$. So we first expand the integrand about $\epsilon = 0$ and then carry out the integration over x .

$$\begin{aligned}
X_1 = & \int_0^1 dy y^{-\epsilon} (1-y)^{-1+\epsilon} \left[\frac{1}{\epsilon} - \frac{2(1-\epsilon)}{\epsilon(1+\epsilon)} y + \frac{2(1-\epsilon)(1-2\epsilon)}{\epsilon(1+\epsilon)} \left\{ \frac{1}{2} y^2 + \frac{5}{4} \epsilon y^2 - \epsilon y \right. \right. \\
& \left. \left. - \epsilon(1-y)^2 \ln(1-y) + \frac{21}{8} \epsilon^2 y^2 - \frac{5}{2} \epsilon^2 y + \dots + 0(\epsilon^3) \right\} \right]. \tag{IV.5}
\end{aligned}$$

Here dots imply the terms of order ϵ^2 , which after integration over y will contribute to order ϵ in X_1 . Since we are using minimal subtraction scheme, those terms of order ϵ in X_1 are not relevant for us. Carrying out the integration over y and retaining terms upto zeroth order in ϵ we obtain

$$X_1 = \frac{1}{\epsilon} + \frac{3}{2}. \tag{IV.6}$$

In a similar fashion we can carry out the integration in X_2 and the result is

$$X_2 = \frac{4}{\epsilon} + 4. \quad (\text{IV.7})$$

Using the result of X_1 and X_2 we can obtain $J_A^{(0)}$.

Using eqn.(III.8) we perform the integration over k and the result is

$$J_A^{(1)} = \frac{3m_A^2}{16\pi^2\epsilon} \left(\frac{m_A}{4\pi\Lambda^2} \right)^{-\epsilon} \left[\mu_A \sqrt{\mu_A^2 - m_A^2} - m_A^2 \ln \left(\frac{\mu_A + \sqrt{\mu_A^2 - m_A^2}}{m_A} \right) \right]. \quad (\text{IV.8})$$

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Figure Captions

Figure 1 : (a) Self energy diagram of quark at order α_s . (b) The counterterm diagram of quark two-point function at order α_s .

Figure 2: (a) Gluon polarisation diagram at order α_s with only the quarks inside the loop. (b) The counterterm diagram of gluon two-point function at order α_s .

Figure 3: Diagrams contributing to the free energy density at order α_s . Here solid lines represent the quarks, coiled lines represent the gluons and dashed lines represent the ghosts.

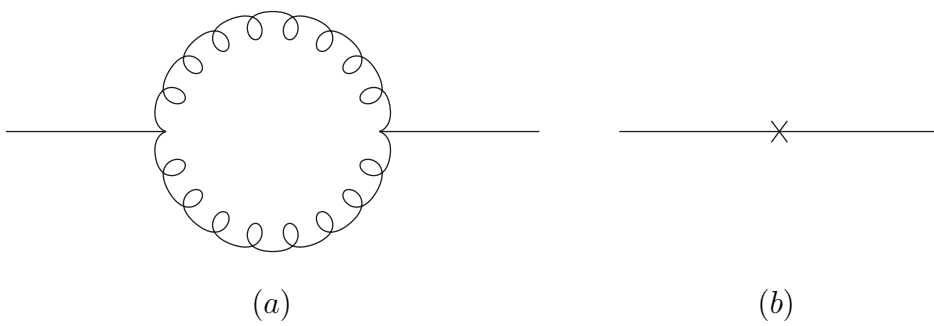


Figure 1.

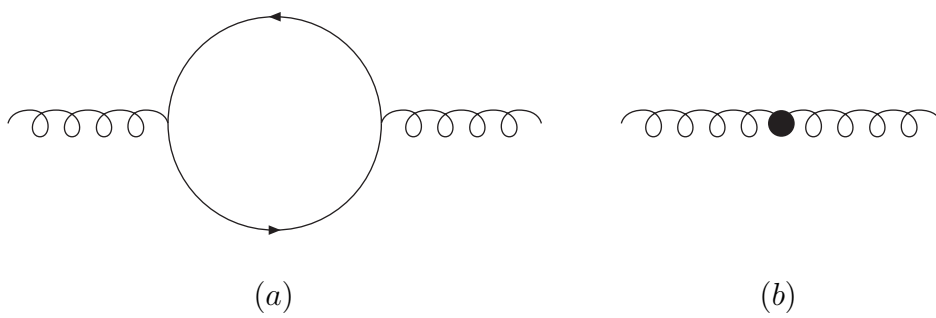
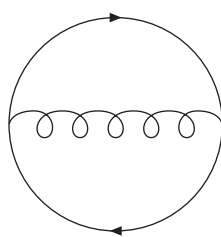
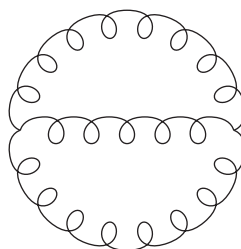


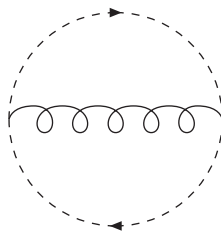
Figure 2.



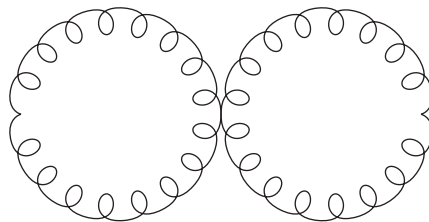
(a)



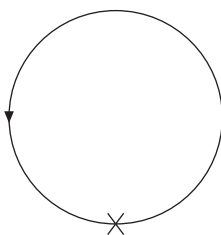
(b)



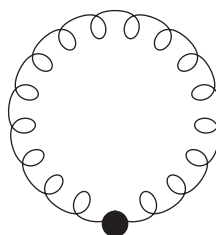
(c)



(d)



(e)



(f)

Figure 3.